AP Calculus BC   WS 79 – DCT & LCT

1) $\sum_{n=1}^{\infty} \frac{1+\cos n}{n^2} = S$

$b_n = \frac{2}{n^2} \quad a_n = \frac{1+\cos n}{n^2}$

$\sum_{n=1}^{\infty} \frac{2}{n^2}$ converges by p-series

Since $b_n \geq a_n$, S converges by DCT

2) $\sum_{n=1}^{\infty} \frac{2n}{3n-1} = S$

$a_n = \frac{2n}{3n-1}$, $\lim_{n\to\infty} a_n = \frac{2}{3} \neq 0$

$\therefore$ S diverges by n^{th} term test

3) $\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}} = S$

$a_n = \frac{n+1}{n^2\sqrt{n}} \quad b_n = \frac{2}{n^{3/2}}$

$\sum_{n=1}^{\infty} \frac{2}{n^{3/2}}$ converges by p-series

n	a_n	b_n
1	2	2
4	5/32	1/4
9	10/243	2/27

Since $b_n \geq a_n$, S converges by DCT

4) $\sum_{n=1}^{\infty} \frac{1}{3^{n-1}+1} = S$

$b_n = \frac{1}{3^{n-1}}$

$\sum_{n=1}^{\infty} \frac{1}{3^{n-1}}$ converges by GST

n	a_n	b_n
1	1/2	1
2	1/4	1/3
3	1/10	1/9

Since $b_n \geq a_n$, S converges by DCT

5) $\sum_{n=1}^{\infty} \frac{3^{n-1}+1}{3^n} = S$

$\lim_{n\to\infty} \frac{3^{n-1}+1}{3^n} = \frac{1}{3} \neq 0$

S diverges by n^{th} term test

6) $\sum_{n=1}^{\infty} \frac{1}{1+\ln n} = S$

$b_n = \frac{1}{n}$

$\sum \frac{1}{n}$ diverges by p-series

n	a_n	b_n
1	1	1
2		1/2
3		1/3

Since $b_n \leq a_n$, S diverges by DCT

7) $\sum_{n=1}^{\infty}\left(\frac{n}{3n+1}\right)^n = S$

$b_n = \left(\frac{1}{3}\right)^n$

$\lim_{n\to\infty}\left[\frac{n^n}{(3n+1)^n}\cdot\frac{3^n}{1}\right]$

$\lim_{n\to\infty}\left[\frac{(3n)^n}{(3n+1)^n}\right] = 1 > 0$

$\sum b_n$ converges by GST

S converges by LCT

8) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^3+2}} = S$

$b_n = \frac{1}{n^{3/2}}$

$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^3+2}}\cdot n^{3/2}\right] = 1 > 0$

$\sum b_n$ converges by p-series

S converges by LCT

9) $\sum_{n=1}^{\infty}\frac{1}{(\ln n)^2} = S$

$b_n = \frac{1}{n}$

$\lim_{n\to\infty}\left[\frac{1}{(\ln n)^2}\cdot n\right]$

$\lim_{n\to\infty}\left[\frac{n}{(\ln n)^2}\right] = \infty$

$\sum b_n$ diverges by p-series

S diverges by LCT

10) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}\ln n} = S$

$b_n = \frac{1}{n}$

$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n}\ln n}\cdot n\right]$

$\lim_{n\to\infty}\left[\frac{n^{1/2}}{\ln n}\right] = \infty$

$\sum b_n$ diverges by p-series

S diverges by LCT

11) $\sum_{n=1}^{\infty}\frac{10n+1}{n(n+1)(n+2)} = S$

$b_n = \frac{10}{n^2}$

$\lim_{n\to\infty}\left[\frac{10n+1}{n(n+1)(n+2)}\cdot\frac{n^2}{10}\right] = 1 > 0$

$\sum_{n=1}^{\infty}\frac{10}{n^2}$ converges by p-series

S converges by LCT

12) $\sum_{n=3}^{\infty}\frac{5n^3-3n}{n^2(n-2)(n^2+5)} = S$

$b_n = \frac{5}{n^2}$

$\lim_{n\to\infty}\left[\frac{5n^3-3n}{n^2(n-2)(n^2+5)}\cdot\frac{n^2}{5}\right] = 1 > 0$

$\sum_{n=1}^{\infty} b_n$ converges by p-series

S converges by LCT